length to the ruler EF be faſtened by one end to the point G, and by the other to the extremity F of the ruler EF. Then if the ſtring be held down to the ruler EF by a pin H, as is repreſented in the figure; the point of this pin, while the ruler EF moves on the ruler CD, ſhall deſcribe the line IKL, which will be one part of the curve line, whoſe deſcription we were here to teach: and by applying the rulers in the like manner on the other ſide of the line AB, we may deſcribe the other part IM of this line. If the diſtance CG be equal to half the line EF in fig. 64, the line MIL will be that very line, wherein the plane ABCD in that figure cuts the cone.

84. The line AI is called the axis of the parabola MIL, and the point G is called the focus.

85. Now by comparing the effects of gravity upon falling bodies, with what is demonſtrated of this figure by the geometers, it is proved, that every body thrown obliquely is carried forward in one of theſe lines, the axis whereof is perpendicular to the horizon.

86. The geometers demonſtrate, that if a line be drawn to touch a parabola in any point, as the line AB (in fig. 68.) touches the parabola CD, whoſe axis is YZ, in the point E; and ſeveral lines FG, HI, KL be drawn parallel to the axis of the parabola: then the line FG will be to HI in the duplicate proportion of EF to EH, and FG to KL in the duplicate proportion of EF to EK; likewiſe HI to KL in the duplicate proportion of EH to EK. What is to be underſtood by duplicate or two-fold

proportion, has been already explained [a]. Accordingly I mean here, that if the line M be taken to bear the ſame proportion to EH, as EH bears to EF, HI will bear the ſame proportion to FG, as M bears to EF; and if the line N bears the ſame proportion to EK, as EK bears to EF, KL will bear the ſame proportion to FG, as N bears to EF; or if the line O bear the ſame proportion to EK, as EK bears to EH, KL will bear the ſame proportion to HI, as O bears to EH.

87. This property is eſſential to the parabola, being ſo connected with the nature of the figure, that every line poſſeſſing this property is to be called by this name.

88. Now ſuppoſe a body to be thrown from the point A (in fig. 69.) towards B in the direction of the line AB. This body, if left to it ſelf, would move on with a uniform motion through this line AB. Suppoſe the eye of a ſpectator to be placed at the point C juſt under the point A; and let us imagine the earth to be ſo put into motion along with the body, as to carry the ſpectator's eye along the line CD parallel to AB; and that the eye ſhould move on with the ſame velocity, wherewith the body would proceed in the line AB, if it were to be left to move without any diſturbance from its gravitation towards the earth. In this caſe if the body moved on without being drawn towards the earth, it would appear to the ſpectator to be at reſt. But if the power of gravity exerted it ſelf on the body, it would appear to the ſpe-

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[a] Chap. 2. § 17.

ctator to fall directly down. Suppoſe at the diſtance of time, wherein the body by its own progreſſive motion would have moved from A to E, it ſhould appear to the ſpectator to have fallen through a length equal to EF: then the body at the end of this time will actually have arrived at the point F. If in the ſpace of time, wherein the body would have moved by its progreſſive motion from A to G, it would have appeared to the ſpectator to have fallen down the ſpace GH: then the body at the end of this greater interval of time will be arrived at the point H. Now if the line AFHI be that, through which the body actually paſſes; from what has here been ſaid, it will follow, that this line is one of thoſe, which I have been deſcribing under the name of the parabola. For the diſtances EF, GH, through which the body is ſeen to fall, will increaſe in the duplicate proportion of the times[a]; but the lines AE, AG will be proportional to the times wherein they would have been deſcribed by the ſingle progreſſive motion of the body: therefore the lines EF, GH will be in the duplicate proportion of the lines AF, AG; and the line AFHI poſſeſſes the property of the parabola.

89. IF the earth be not ſuppoſed to move along with the body, the caſe will be a little different. For the body being conſtantly drawn directly towards the center of the earth, the body in its motion will be drawn in a direction a little oblique to that, wherein it would be drawn by the earth in motion, as before ſuppoſed. But the diſtance to the center of the

earth bears ſo vaſt a proportion to the greateſt length, to which we can throw bodies, that this obliquity does not merit any regard. From the ſequel of this diſcourſe it may indeed be collected, what line the body being thrown thus would be found to deſcribe, allowance being made for this obliquity of the earth's action[a]. This is the diſcovery of Sir Is. Newton; but has no uſe in this place. Here it is abundantly ſufficient to conſider the body as moving in a parabola.

90. The line, which a projected body deſcribes, being thus known, practical methods have been deduced from hence for directing the ſhot of great guns to ſtrike any object deſired. This work was firſt attempted by Galileo, and ſoon after farther improved by his ſcholar Torricelli; but has lately been rendred more complete by the great Mr. Cotes, whoſe immature death is an unſpeakable loſs to mathematical learning. If it be required to throw a body from the point A (in fig. 70.) ſo as to ſtrike the point B; through the points A, B draw the ſtraight line CD, and erect the line AE perpendicular to the horizon, and of four times the height, from which a body muſt fall to acquire the velocity, wherewith the body is intended to be thrown. Through the points A and E deſcribe a circle, that ſhall touch the line CD in the point A. Then from the point B draw the line BF perpendicular to the horizon, interſecting the circle in the points G and H. This being done, if the body be projected directly towards either of theſe points G or H, it ſhall fall upon the point B; but with this difference, that, if it be thrown

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[a] From B II. Ch. 3.

in the direction AG, it ſhall ſooner arrive at B, than if it were projected in the direction AH. When the body is projected in the direction AG; the time, it will take up in arriving at B, will bear the ſame proportion to the time wherein it would fall down through one fourth part of AE, as AG bears to half AE. But when the body is thrown in the direction of AH, the time of its paſſing to B will bear the ſame proportion to the time, wherein it would fall through one fourth part of AE, as AH bears to half AE.

91. IF the line AI be drawn ſo as to divide the angle under EAD in the middle, and the line IK be drawn perpendicular to the horizon; this line will touch the circle in the point I, and if the body be thrown in the direction AI, it will fall upon the point K: and this point K is the fartheſt point in the line AD, which the body can be made to ſtrike, without increaſing its velocity.

92. THE velocity, wherewith the body every where moves, may be found thus. Suppoſe the body to move in the parabola AB (fig. 71.) Erect AC perpendicular to the horizon, and equal to the height, from which a body muſt fall to acquire the velocity, wherewith the body ſets out from A. If you take any points as D and E in the parabola, and draw DF and EG parallel to the horizon; the velocity of the body in D will be equal to what a body will acquire in falling down by its own weight through CF, and in E the velocity will be the ſame, as would be acquired in falling through CG. Thus the body moves ſloweſt at the higheſt point H of the parabola; and at equal diſtances from this point will

move with equal ſwiftneſs, and deſcend from that higheſt point through the line HB altogether like to the line AH in which it aſcended; abating only the reſiſtance of the air, which is not here conſidered. If the line HI be drawn from the higheſt point H parallel to the horizon, AI will be equal to ¼ of BG in fig. 70, when the body is projected in the direction AG, and equal to ¼ of BH, when the body is thrown in the direction AH, provided AD be drawn horizontally.

93. Thus I have recounted the principal diſcoveries, which had been made concerning the motion of bodies by Sir Isaac Newton's predeceſſors; all theſe diſcoveries, by being found to agree with experience, contributing to eſtabliſh the laws of motion, from whence they were deduced. I ſhall therefore here finiſh what I had to ſay upon thoſe laws; and conclude this chapter with a few words concerning the diſtinction which ought to be made between abſolute and relative motion. For ſome have thought fit to confound them together; becauſe they obſerve the laws of motion to take place here on the earth, which is in motion, after the ſame manner as if it were at reſt. But Sir Isaac Newton has been careful to diſtinguiſh between the relative and abſolute conſideration both of motion and time[a]. The aſtronomers anciently found it neceſſary to make this diſtinction in time. Time conſidered in it ſelf paſſes on equably without relation to any thing external, being the proper meaſure of the continuance and duration of all things. But it is moſt frequently conceived of by us under a relative view to ſome ſucceſſion in

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[a] Prin. Philoſ. pag. 7, &c.

ſenſible things, of which we take cognizance. The ſucceſſion of the thoughts in our own minds is that, from whence we receive our firſt idea of time, but is a very uncertain meaſure thereof; for the thoughts of ſome men flow on much more ſwiftly, than the thoughts of others; nor does the ſame perſon think equally quick at all times. The motions of the heavenly bodies are more regular; and the eminent diviſion of time into night and day, made by the ſun, leads us to meaſure our time by the motion of that luminary: nor do we in the affairs of life concern our ſelves with any inequality, which there may be in that motion; but the ſpace of time which comprehends a day and night is rather ſuppoſed to be always the ſame. However aſtronomers anciently found theſe ſpaces of time not to be always of the ſame length, and have taught how to compute their differences. Now the time, when ſo equated as to be rendered perfectly equal, is the true meaſure of duration, the other not. And therefore this latter, which is abſolutely true time, differs from the other, which is only apparent. And as we ordinarily make no diſtinction between apparent time, as meaſured by the ſun, and the true; ſo we often do not diſtinguiſh in our uſual diſcourſe between the real, and the apparent or relative motion of bodies; but uſe the ſame words for one, as we ſhould for the other. Though all things about us are really in motion with the earth; as this motion is not viſible, we ſpeak of the motion of every thing we ſee, as if our ſelves and the earth ſtood ſtill. And even in other caſes, where we diſcern the motion of bodies, we often ſpeak of them not in relation to the whole motion we ſee, but with regard to other bodies, to which they are contiguous. If any body were lying on a table; when that table ſhall be carried along, we ſay the body reſts upon the table, or perhaps abſolutely, that the body is at reſt. However philoſophers muſt not reject all diſtinction between true and apparent motions, any more than aſtronomers do the diſtinction between true and vulgar time; for there is as real a difference between them, as will appear by the following conſideration. Suppoſe all the bodies of the univerſe to have their courſes ſtopped, and reduced to perfect reſt. Then ſuppoſe their preſent motions to be again reſtored; this cannot be done without an actual impreſſion made upon ſome of them at leaſt. If any of them be left untouched, they will retain their former ſtate, that is, ſtill remain at reſt; but the other bodies, which are wrought upon, will have changed their former ſtate of reſt, for the contrary ſtate of motion. Let us now ſuppoſe the bodies left at reſt to be annihilated, this will make no alteration in the ſtate of the moving bodies; but the effect of the impreſſion, which was made upon them, will ſtill ſubſiſt. This ſhews the motion they received to be an abſolute thing, and to have no neceſſary dependence upon the relation which the body ſaid to be in motion has to any other body [a].

94. BESIDES abſolute and relative motion are diſtinguiſhable by their Effects. One effect of motion is, that bodies, when moved round any center or axis, acquire a certain

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[a] See Newton. princip. philoſ. pag. 9. lin. 30.

power, by which they forcibly preſs themſelves from that center or axis of motion. As when a body is whirled about in a ſling, the body preſſes againſt the ſling, and is ready to fly out as ſoon as liberty is given it. And this power is proportional to the true, not relative motion of the body round ſuch a center or axis. Of this Sir Isaac Newton gives the following inſtance [a]. If a pail or ſuch like veſſel near full of water be ſuſpended by a ſtring of ſufficient length, and be turned about till the ſtring be hard twiſted. If then as ſoon as the veſſel and water in it are become ſtill and at reſt, the veſſel be nimbly turned about the contrary way the ſtring was twiſted, the veſſel by the ſtrings untwiſting it ſelf ſhall continue its motion a long time. And when the veſſel firſt begins to turn, the water in it ſhall receive little or nothing of the motion of the veſſel, but by degrees ſhall receive a communication of motion, till at laſt it ſhall move round as ſwiftly as the veſſel it ſelf. Now the definition of motion, which Des Cartes has given us upon this principle of making all motion meerly relative, is this: that motion, is a removal of any body from its vicinity to other bodies, which were in immediate contact with it, and are conſidered as at reſt [b]. And if this be compared with what he ſoon after ſays, that there is nothing real or poſitive in the body moved, for the ſake of which we aſcribe motion to it, which is not to be found as well in the contiguous bodies, which are conſidered as at reſt [c]; it will follow from thence, that we may conſider the veſſel as at reſt and the water as moving in it: and the water in reſpect of the veſſel has the greateſt motion, when the veſſel firſt begins to turn, and loſes this relative motion more and more, till at length it quite ceaſes. But now, when the veſſel firſt begins to turn, the ſurface of the water remains ſmooth and flat, as before the veſſel began to move; but as the motion of the veſſel communicates by degrees motion to the water, the ſurface of the water will be obſerved to change, the water ſubſiding in the middle and riſing at the edges: which elevation of the water is cauſed by the parts of it preſſing from the axis, they move about; and therefore this force of receding from the axis of motion depends not upon the relative motion of the water within the veſſel, but on its abſolute motion; for it is leaſt, when that relative motion is greateſt, and greateſt, when that relative motion is leaſt, or none at all.

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[a] Princip. Philoſ. pag. 10.

[b] Renati Des Cart. Princ. Philoſ. Part. II. § 25.

Ibid. § 30.

95. THUS the true cauſe of what appears in the ſurface of this water cannot be aſſigned, without conſidering the water's motion within the veſſel. So alſo in the ſyſtem of the world, in order to find out the cauſe of the planetary motions, we muſt know more of the real motions, which belong to each planet, than is abſolutely neceſſary for the uſes of aſtronomy. If the aſtronomer ſhould ſuppoſe the earth to ſtand ſtill, he could aſcribe ſuch motions to the celeſtial bodies, as ſhould anſwer all the appearances; though he would not account for them in ſo ſimple a manner, as by attributing motion to the earth. But the motion of the earth muſt of neceſſity be conſidered, before the real cauſes, which actuate the planetary ſyſtem, can be diſcovered.

CHAP. III.

Of CENTRIPETAL FORCES.

WE have juſt been deſcribing in the preceding chapter the effects produced on a body in motion, from its being continually acted upon by a power always equal in ſtrength, and operating in parallel directions [a]. But bodies may be acted upon by powers, which in different places ſhall have different degrees of force, and whoſe ſeveral directions ſhall be variouſly inclined to each other. The moſt ſimple of theſe in reſpect to direction is, when the power is pointed conſtantly to one center. This is truly the caſe of that power, whoſe effects we deſcribed in the foregoing chapter; though the center of that power is ſo far removed, that the ſubject then before us is moſt conveniently to be conſidered in the light, wherein we have placed it: But Sir ISAAC NEWTON has conſidered very particularly this other caſe of powers, which are conſtantly directed to the ſame center. It is upon this foundation, that all his diſcoveries in the ſyſtem of the world are raiſed. And therefore, as this ſubject bears ſo very great a ſhare in the philoſophy, of which I am diſcourſing, I think it proper in this place to take a ſhort view of ſome of the general effects of theſe powers, before we come to apply them particularly to the ſyſtem of the world.

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[a] § 85, &c.

2. THESE powers or forces are by Sir ISAAC NEWTON called centripetal; and their firſt effect is to cauſe the body, upon which they act, to quit the ſtraight courſe, wherein it would proceed if undiſturbed, and to deſcribe an incurvated line, which ſhall always be bent towards the center of the force. It is not neceſſary, that ſuch a power ſhould cauſe the body to approach that center. The body may continue to recede from the center of the power, notwithſtanding its being drawn by the power; but this property muſt always belong to its motion, that the line, in which it moves, will continually be concave towards the center, to which the power is directed. Suppoſe A (in fig. 72.) to be the center of a force. Let a body in B be moving in the direction of the ſtraight line BC, in which line it would continue to move, if undiſturbed; but being attracted by the centripetal force towards A, the body muſt neceſſarily depart from this line BC, and being drawn into the curve line BD, muſt paſs between the lines AB and BC. It is evident therefore, that the body in B being gradually turned off from the ſtraight line BC, it will at firſt be convex toward the line BC, and conſequently concave towards the point A: for theſe centripetal powers are ſuppoſed to be in ſtrength proportional to the power of gravity, and, like that, not to be able after the manner of an impulſe to turn the body ſenſibly out of its courſe into a different one in an inſtant, but to take up ſome ſpace of time in producing a viſible effect. That the curve will always continue to have its concavity towards A may thus appear. In the line BC near to B take any point as E, from which the line EFG may be ſo

drawn, as to touch the curve line B D in ſome point as F. Now when the body is come to F, if the centripetal power were immediately to be ſuſpended, the body would no longer continue to move in a curve line, but being left to it ſelf would forthwith reaſſume a ſtraight courſe; and that ſtraight courſe would be in the line F G: for that line is in the direction of the body's motion at the point F. But the centripetal force continuing its energy, the body will be gradually drawn from this line F G ſo as to keep in the line F D, and make that line near the point F to be convex toward F G, and concave toward A. After the ſame manner the body may be followed on in its courſe through the line B D, and every part of that line be ſhewn to be concave toward the point A.

3. THIS then is the conſtant character belonging to thoſe motions, which are carried on by centripetal forces; that the line, wherein the body moves, is throughout concave towards the center of the force. In reſpect to the ſucceſſive diſtances of the body from the center there is no general rule to be laid down; for the diſtance of the body from the center may either increaſe, or decreaſe, or even keep always the ſame. The point A (in fig. 73.) being the center of a centripetal force, let a body at B ſet out in the direction of the ſtraight line B C perpendicular to the line A B drawn from A to B. It will be eaſily conceived, that there is no other point in the line B C ſo near to A, as the point B; that A B is the ſhorteſt of all the lines, which can be drawn from A to any part of the line B C; all other lines, as A D, or A E, drawn from A to the line B C being longer than A B. Hence it follows, that the body ſet-

ting out from B, if it moved in the line BC, it would recede more and more from the point A. Now as the operation of a centripetal force is to draw a body towards the center of the force: if ſuch a force act upon a reſting body, it muſt neceſſarily put that body ſo into motion, as to cauſe it to move towards the center of the force: if the body were of it ſelf moving towards that center, the centripetal force would accelerate that motion, and cauſe it to move faſter down: but if the body were in ſuch a motion, as being left to itſelf it would recede from this center, it is not neceſſary, that the action of a centripetal power upon it ſhould immediately compel the body to approach the center, from which it would otherwiſe have receded; the centripetal power is not without effect, if it cauſe the body to recede more ſlowly from that center, than otherwiſe it would have done. Thus in the caſe before us, the ſmalleſt centripetal power, if it act on the body, will force it out of the line BC, and cauſe it to paſs in a bent line between BC and the point A, as has been before explained. When the body, for inſtance, has advanced to the line AD, the effect of the centripetal force diſcovers it ſelf by having removed the body out of the line BC, and brought it to croſs the line AD ſomewhere between A and D: ſuppoſe at F. Now AD being longer than AB, AF may alſo be longer than AB. The centripetal power may indeed be ſo ſtrong, that AF ſhall be ſhorter than AB; or it may be ſo evenly balanced with the progreſſive motion of the body, that AF and AB ſhall be juſt equal: and in this laſt caſe, when the centripetal force is of that ſtrength, as conſtantly to draw the body as much toward the center, as the progreſſive motion would carry it off, the body will deſcribe a circle about the center A, this center of the force being alſo the center of the circle.

4. IF the body, inſtead of ſetting out in the line BC perpendicular to AB, had ſet out in another line BG more inclined towards the line AB, moving in the curve line BH; then as the body, if it were to continue its motion in the line BG, would for ſome time approach the center A; the centripetal force would cauſe it to make greater advances toward that center. But if the body were to ſet out in the line BI reclined the other way from the perpendicular BC, and were to be drawn by the centripetal force into the curve line BK; the body, notwithſtanding any centripetal force, would for ſome time recede from the center; ſince ſome part at leaſt of the curve line BK lies between the line BI and the perpendicular BC.

5. THUS far we have explained ſuch effects, as attend every centripetal force. But as theſe forces may be very different in regard to the different degrees of ſtrength, wherewith they act upon bodies in different places; I ſhall now proceed to make mention in general of ſome of the differences attending theſe centripetal motions.

6. TO reaſſume the conſideration of the laſt mentioned caſe. Suppoſe a centripetal power directed toward the point A (in fig. 74.) to act on a body in B, which is moving in the direction of the ſtraight line BC, the line BC reclining off from AB. If from A the ſtraight lines AD, AE, AF are

drawn at pleaſure to the line C B; the line C B being prolonged beyond B to G, it appears that A D is inclined to the line G C more obliquely, than A B is inclined to it, A E is inclined more obliquely than A D, and A F more than A E. To ſpeak more correctly, the angle under A D G is leſs than that under A B G, the angle under A E G leſs than that under A D G, and the angle under A F G leſs than that under A E G. Now ſuppoſe the body to move in the curve line B H I K. Then it is here likewiſe evident, that the line B H I K being concave towards A, and convex towards the line B C, it is more and more turned off from the line B C; ſo that in the point H the line A H will be leſs obliquely inclined to the curve line B H I K, than the ſame line A H D is inclined to B C at the point D; at the point I the inclination of the line A I to the curve line will be more different from the inclination of the ſame line A I E to the line B C, at the point E; and in the points K and F the difference of inclination will be ſtill greater; and in both the inclination at the curve will be leſs oblique, than at the ſtraight line B C. But the ſtraight line A B is leſs obliquely inclined to B G, than A D is inclined towards D G: therefore although the line A H be leſs obliquely inclined towards the curve H B, than the ſame line A H D is inclined towards D G; yet it is poſſible, that the inclination at H may be more oblique, than the inclination at B. The inclination at H may indeed be leſs oblique than the other, or they may be both the ſame. This depends upon the degree of ſtrength, wherewith the centripetal force exerts it ſelf, during the paſſage of the body from B to H. After the ſame manner the inclinations at I and K depend entirely on the degree

gree of ſtrength, wherewith the centripetal force acts on the body in its paſſage from H to K: if the centripetal force be weak enough, the lines AH and AI drawn from the center A to the body at H and at I ſhall be more obliquely inclined to the curve, than the line AB is inclined towards BG. The centripetal force may be of that ſtrength as to render all theſe inclinations equal, or if ſtronger, the inclinations at I and K will be leſs oblique than at B. Sir ISAAC NEWTON has particularly ſhewn, that if the centripetal power decreaſes after a certain manner with the increaſe of diſtance, a body may deſcribe ſuch a curve line, that all the lines drawn from the center to the body ſhall be equally inclined to that curve line [a]. But I do not here enter into any particulars, my preſent intention being only to ſhew, that it is poſſible for a body to be acted upon by a force continually drawing it down towards a center, and yet that the body ſhall continue to recede from that center; for here as long as the lines AH, AI, &c drawn from the center A to the body do not become leſs oblique to the curve, in which the body moves; ſo long ſhall thoſe lines perpetually increaſe, and conſequently the body ſhall more and more recede from the center.

7. BUT we may obſerve farther, that if the centripetal power, while the body increaſes its diſtance from the center, retain ſufficient ſtrength to make the lines drawn from the center to the body to become at length leſs oblique to the curve; then if this diminution of the obliquity continue, till

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[a] Princip. Philoſ. Lib. I. prop. 9.

at laſt the line drawn from the center to the body ſhall ceaſe to be obliquely inclined to the curve, and ſhall become perpendicular thereto; from this inſtant the body ſhall no longer recede from the center, but in its following motion it ſhall again deſcend, and ſhall deſcribe a curve line in all reſpects like to that, which it has deſcribed already; provided the centripetal power, every where at the ſame diſtance from the center, acts with the ſame ſtrength. So we obſerved in the preceding chapter, that, when the motion of a projectile became parallel to the horizon, the projectile no longer aſcended, but forthwith directed its courſe downwards, deſcending in a line altogether like that, wherein it had before aſcended[a].

8. THIS return of the body may be proved by the following propoſition: that if the body in any place, ſuppoſe at I, were to be ſtopt, and be thrown directly backward with the velocity, wherewith it was moving forward in that point I; then the body, by the action of the centripetal force upon it, would move back again over the path IHB, in which it had before advanced forward, and would arrive again at the point B in the ſame ſpace of time, as was taken up in its paſſage from B to I; the velocity of the body at its return to the point B being the ſame, as that wherewith it firſt ſet out from that point. To give a full demonſtration of this propoſition, would require that uſe of mathematics, which I here purpoſe to avoid; but, I believe, it will appear in great meaſure evident from the following conſiderations.

[a] § 92.

9. SUPPOSE (in fig. 75.) that a body were carried after the following manner through the bent figure ABCDEF; compoſed of the ſtraight lines AB, BC, CD, DE, EF. Firſt let it be moving in the line AB, from A towards B, with any uniform velocity. At B let the body receive an impulſe directed toward ſome point, as G, taken within the concavity of the figure. Now whereas this body, when once moving in the ſtraight line AB, will continue to move on in this line, ſo long as it ſhall be left to it ſelf; but being diſturbed at the point B in its motion by the impulſe, which there acts upon it, it will be turned out of this line AB into ſome other ſtraight line, wherein it will afterwards continue to move, as long as it ſhall be left to itſelf. Therefore let this impulſe have ſtrength ſufficient to turn the body into the line BC. Then let the body move on undiſturbed from B to C, but at C let it receive another impulſe pointed toward the ſame point G, and of ſufficient ſtrength to turn the body into the line CD. At D let a third impulſe, directed like the reſt to the point G, turn the body into the line DE. And at E let another impulſe, directed likewiſe to the point G, turn the body into the line EF. Now, I ſay, if the body while moving in the line EF be ſtopt, and turned back again in this line with the ſame velocity, as that wherewith it was moving forward in this line; then by the repetition of the former impulſe at E the body will be turned into the line ED, and move in it from E to D with the ſame velocity as before it moved with from D to E; by the repetition of the impulſe at D, when the body ſhall have returned to that point, it will be turned into the line DC; and by the repetition of the other impulſes at C and B

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the body will be brought back again into the line B A, with the velocity, wherewith it firſt moved in that line.

10. THIS I prove as follows. Let D E and F E be continued beyond E. In D E thus continued take at pleaſure the length E H, and let H I be ſo drawn, as to be equidiſtant from the line G E. Then, by what has been written upon the ſecond law of motion [a], it follows, that after the impulſe on the body in E it will move through E I in the ſame time, as it would have imployed in moving from E to H, with the velocity which it had in the line D E. In F E prolonged take E K equal to E I, and draw K L equidiſtant from G E. Then, becauſe the body is thrown back in the line F E with the ſame velocity as that wherewith it went forward in that line; if, when the body was returned to E, it were permitted to go ſtraight on, it would paſs through E K in the ſame time, as it took up in paſſing through E I, when it went forward in the line E F. But, if at the body's return to the point E, ſuch an impulſe directed toward the point D were to be given it, whereby it ſhould be turned into the line D E; I ſay, that the impulſe neceſſary to produce this effect muſt be equal to that, which turned the body out of the line D E into E F; and that the velocity, with which the body will return into the line E D, is the ſame, as that wherewith it before moved through this line from D to E. Becauſe E K is equal to E I, and K L and H I, being each equidiſtant from G E, are by conſequence equidiſtant from each other; it follows, that the two

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[a] Ch. II. § 22.

triangular figures IEH and KEL are altogether like and equal to each other. If I were writing to mathematicians, I might refer them to ſome propoſitions in the elements of EUCLID for the proof of this [a]: but as I do not here addreſs my ſelf to ſuch, ſo I think this aſſertion will be evident enough without a proof in form; at leaſt I muſt deſire my readers to receive it as a propoſition true in geometry. But theſe two triangular figures being altogether like each other and equal; as EK is equal to EI, ſo EL is equal to EH, and KL equal to HI. Now the body after its return to E being turned out of the line FE into ED by an impulſe acting upon it in E, after the manner above expreſſed; the body will receive ſuch a velocity by this impulſe, as will carry it through EL in the ſame time, as it would have imployed in paſſing through EK, if it had gone on in that line undiſturbed. And it has already been obſerved, that the time, in which the body would paſs over EK with the velocity wherewith it returns, is equal to the time it took up in going forward from E to I; that is, equal to the time, in which it would have gone through EH with the velocity, wherewith it moved from D to E. Therefore the time, in which the body will paſs through EL after its return into the line ED, is the ſame, as would have been taken up by the body in paſſing through EH with the velocity, wherewith the body firſt moved in the line DE. Since therefore EL and EH are equal, the body returns into the line DE with the velocity, which it had before in that line. Again I ſay, the ſecond impulſe in E is equal to the firſt. By what has been ſaid on the ſecond law of motion concerning the effect of oblique impulſes[a], it will be underſtood, that the impulſe in E, whereby the body was turned out of the line DE into the line EF, is of ſuch ſtrength, that if the body had been at reſt, when this impulſe had acted upon it, this impulſe would have communicated ſo much motion to the body, as would have carried it through a length equal to HI, in the time wherein the body would have paſſed from E to H, or in the time wherein it paſſed from E to I. In the ſame manner, on the return of the body, the impulſe in E, whereby the body is turned out of the line FE into ED, is of ſuch ſtrength, that if it had acted on the body at reſt, it would have cauſed the body to move through a length equal to KL, in the ſame time, as the body would imploy in paſſing through EK with the velocity, wherewith it returns in the line FE. Therefore the ſecond impulſe, had it acted on the body at reſt, would have cauſed it to move through a length equal to KL in the ſame ſpace of time, as would be taken up by the body in paſſing through a length equal to HI, were the firſt impulſe to act on the body when at reſt. That is, the effects of the firſt and ſecond impulſe on the body when at reſt would be the ſame; for KL and HI are equal: conſequently the ſecond impulſe is equal to the firſt.

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[a] Viz. L. I. prop. 30, 29, & 26.

11. Thus if the body be returned through FE with the velocity, wherewith it moved forward; we have ſhewn how by the repetition of the impulſe, which acted on it at E, the

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[a] Ch. II. §21, 22.

body will return again into the line DE with the velocity, which it had before in that line. By the ſame proceſs of reaſoning it may be proved, that, when the body is returned back to D, the impulſe, which before acted on the body at that point, will throw the body into the line DC with the velocity, which it firſt had in that line; and the other impulſes being ſucceſſively repeated, the body will at length be brought back again into the line BA with the velocity, wherewith it ſet out in that line.

12. THUS theſe impulſes, by acting over again in an inverted order all their operation on the body, bring it back again through the path, in which it had proceeded forward. And this obtains equally, whatever be the number of the ſtraight lines, whereof this curve figure is compoſed. Now by a method of reaſoning, which Sir ISAAC NEWTON makes great uſe of, and which he introduced into geometry, thereby greatly inriching that ſcience [a]; we might make a tranſition from this figure compoſed of a number of ſtraight lines to a figure of one continued curvature, and from a number of ſeparate impulſes repeated at diſtinct intervals to a continual centripetal force, and ſhew, that, becauſe what has been here advanced holds univerſally true, whatever be the number of ſtraight lines, whereof the curve figure ACF is compoſed, and howſoever frequently the impulſes at the angles of this figure are repeated; therefore the ſame will ſtill remain true, although this figure ſhould be converted into one of a continued curvature, and theſe diſtinct impulſes ſhould be

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[a] viz. His doctrine of prime and ultimate ratios.

changed into a continual centripetal force. But as the explaining this method of reaſoning is foreign to my preſent deſign; ſo I hope my readers, after what has been ſaid, will find no difficulty in receiving the propoſition laid down above: that, if the body, which has moved through the curve line B H I (in fig. 74.) from B to I, when it is come to I, be thrown directly back with the ſame velocity as that, wherewith it proceeded forward, the centripetal force, by acting over again all its operation on the body, ſhall bring the body back again in the line I H B: and as the motion of the body in its courſe from B to I was every where in ſuch a manner oblique to the line drawn from the center to the body, that the centripetal power acted in ſome degree againſt the body's motion, and gradually diminiſhed it; ſo in the return of the body, the centripetal power will every where draw the body forward, and accelerate its motion by the ſame degrees, as before it retarded it.

13. This being agreed, ſuppoſe the body in K to have the line A K no longer obliquely inclined to its motion. In this caſe, if the body be turned back, in the manner we have been conſidering, it muſt be directed back perpendicularly to A K. But if it had proceeded forward, it would likewiſe have moved in a direction perpendicular to A K; conſequently, whether it move from this point K backward or forward, it muſt deſcribe the ſame kind of courſe. Therefore ſince by being turned back it will go over again the line K I H B; if it be permitted to go forward, the line K L, which it ſhall deſcribe, will be altogether ſimilar to the line K H B.

14. IN like manner we may determine the nature of the motion, if the line, wherein the body ſets out, be inclined (as in fig. 76.) down toward the line BA drawn between the body and the center. If the centripetal power ſo much increaſes in ſtrength, as the body approaches, that it can bend the path, in which the body moves, to that degree, as to cauſe all the lines as AH, AI, AK to remain no leſs oblique to the motion of the body, than AB is oblique to BC; the body ſhall continually more and more approach the center. But if the centripetal power increaſes in ſo much leſs a degree, as to permit the line drawn from the center to the body, as it accompanies the body in its motion, at length to become more and more erect to the curve wherein the body moves, and in the end, ſuppoſe at K, to become perpendicular thereto; from that time the body ſhall riſe again. This is evident from what has been ſaid above; becauſe for the very ſame reaſon here alſo the body ſhall proceed from the point K to deſcribe a line altogether ſimilar to the line, in which it has moved from B to K. Thus, as it was obſerved of the pendulum in the preceding chapter[a], that all the time it approaches towards being perpendicular to the horizon, it more and more deſcends; but, as ſoon as it is come into that perpendicular ſituation, it immediately riſes again by the ſame degrees, as it deſcended by before: ſo here the body more and more approaches the center all the time it is moving from B to K; but thence forward it riſes from the center again by the ſame degrees, as it approached by before.

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[a] § 57.

15. If (in fig. 77.) the line B C be perpendicular to A B; then it has been obſerved above [a], that the centripetal power may be ſo balanced with the progreſſive motion of the body, that the body may keep moving round the center A conſtantly at the ſame diſtance; as a body does, when whirled about any point, to which it is tyed by a ſtring. If the centripetal power be too weak to produce this effect, the motion of the body will preſently become oblique to the line drawn from itſelf to the center, after the manner of the firſt of the two caſes, which we have been conſidering. If the centripetal power be ſtronger, than what is required to carry the body in a circle, the motion of the body will preſently fall in with the ſecond of the caſes, we have been conſidering.

16. If the centripetal power ſo change with the change of diſtance, that the body, after its motion has become oblique to the line drawn from itſelf to the center, ſhall again become perpendicular thereto; which we have ſhewn to be poſſible in both the caſes treated of above; then the body ſhall in its ſubſequent motion return again to the diſtance of A B, and from that diſtance take a courſe ſimilar to the former: and thus, if the body move in a ſpace free from all reſiſtance, which has been here all along ſuppoſed; it ſhall continue in a perpetual motion about the center, deſcending and aſcending alternately therefrom. If the body ſetting out from B (in fig. 78.) in the line B C perpendicular to A B, deſcribe the line B D E, which in D ſhall be oblique to the line A D, but in E

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[a] § 3.

ſhall again become erect to AE drawn from the body in E to the center A; then from this point E the body ſhall deſcribe the line EFG altogether like to the line BDE, and at G ſhall be at the ſame diſtance from A, as it was at B. But likewiſe the line AG ſhall be erect to the body's motion. Therefore the body ſhall proceed to deſcribe from G the line GHI altogether ſimilar to the line GFE, and at I have the ſame diſtance from the center, as it had at E; and alſo have the line AI erect to its motion: ſo that its following motion muſt be in the line IKL ſimilar to IHG, and the diſtance AL equal to AG. Thus the body will go on in a perpetual round without ceaſing, alternately inlarging and contracting its diſtance from the center.

17. IF it ſo happen, that the point E fall upon the line BA continued beyond A; then the point G will fall on B, I on E, and L alſo on B; ſo that the body will deſcribe in this caſe a ſimple curve line round the center A, like the line BDEF in fig. 79, in which it will continually revolve from B to E and from E to B without end.

18. IF AE in fig. 78 ſhould happen to be perpendicular to AB, in this caſe alſo a ſimple line will be deſcribed; for the point G will fall on the line BA prolonged beyond A, the point I on the line AE prolonged beyond A, and the point L on B: ſo that the body will deſcribe a line like the curve line BEGI in fig. 80, in which the oppoſite points B and G are equally diſtant from A, and the oppoſite points E and I are alſo equally diſtant from the ſame point A.

19. In other caſes the line deſcribed will have a more complex figure.

20. Thus we have endeavoured to ſhew how a body, while it is conſtantly attracted towards a center, may notwithſtanding by its progreſſive motion keep it ſelf from falling down to that center; but deſcribe about it an endleſs circuit, ſometimes approaching toward that center, and at other times as much receding from the ſame.

21. But here we have ſuppoſed, that the centripetal power is of equal ſtrength every where at the ſame diſtance from the center. And this is the caſe of that centripetal power, which will hereafter be ſhewn to be the cauſe, that keeps the planets in their courſes. But a body may be kept on in a perpetual circuit round a center, although the centripetal power have not this property. Indeed a body may by a centripetal force be kept moving in any curve line whatever, that ſhall have its concavity turned every where towards the center of the force.

22. To make this evident I ſhall firſt propoſe the caſe of a body moving through the incurvated figure ABCDE (in fig. 81.) which is compoſed of the ſtraight lines A B, B C, C D, D E, and E A; the motion being carried on in the following manner. Let the body firſt move in the line A B with any uniform velocity. When it is arrived at the point B, let it receive an impulſe directed toward any point F taken within the figure; and let the impulſe be of that ſtrength as to turn the body out

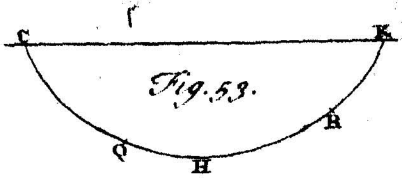
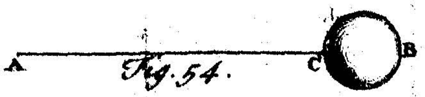


Fig. 53.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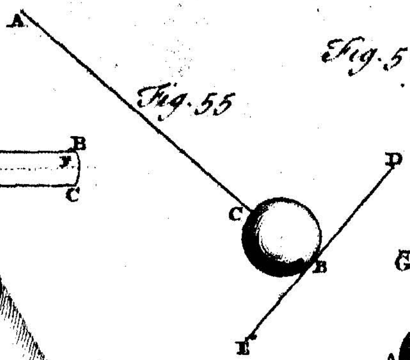
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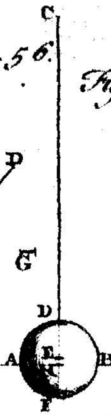
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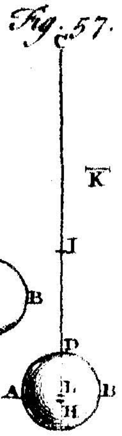
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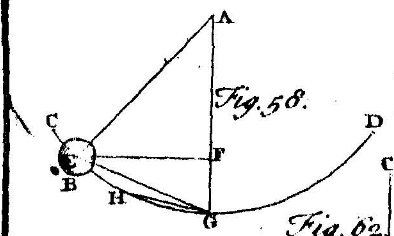
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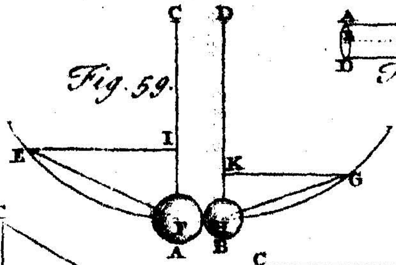
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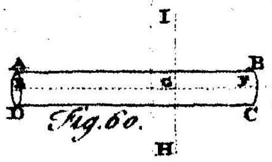
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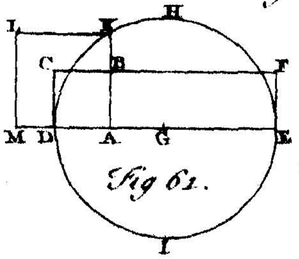
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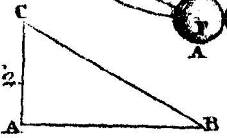
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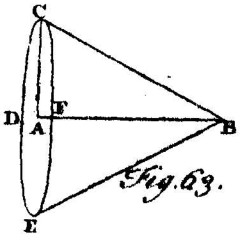
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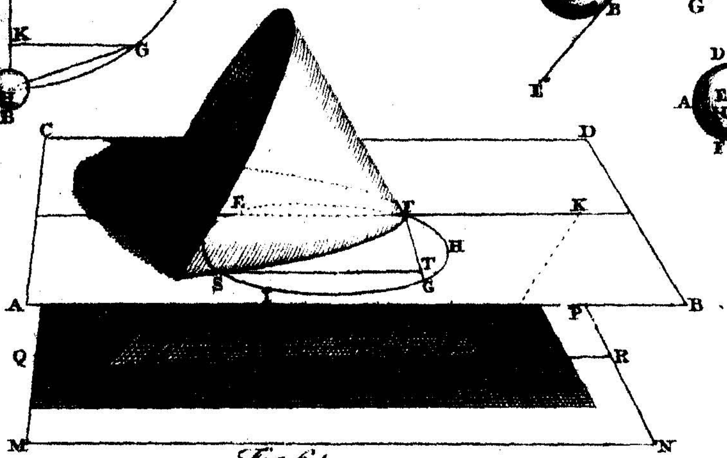
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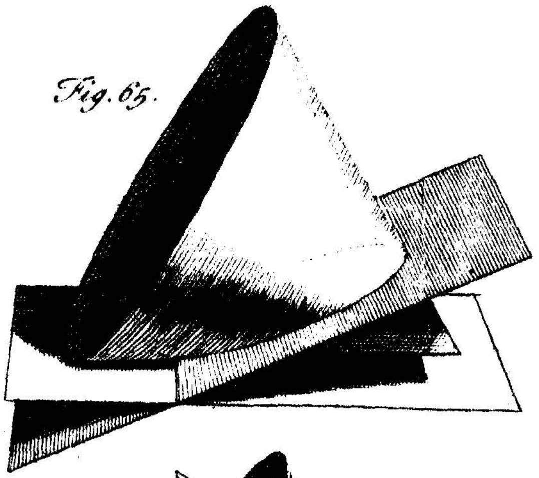
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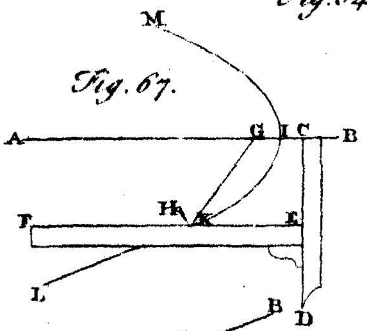
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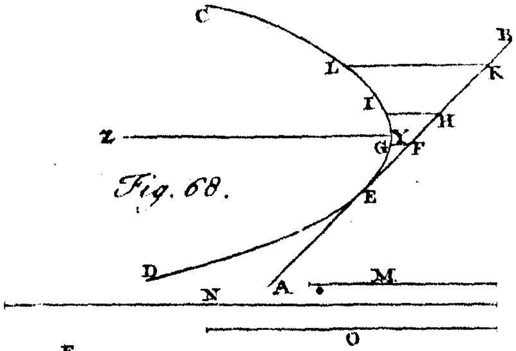
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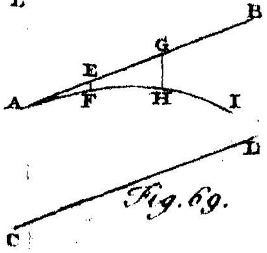
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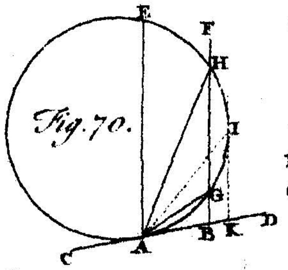
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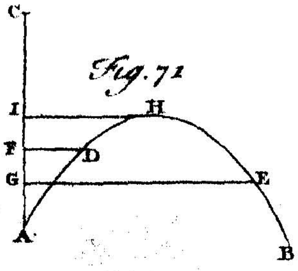
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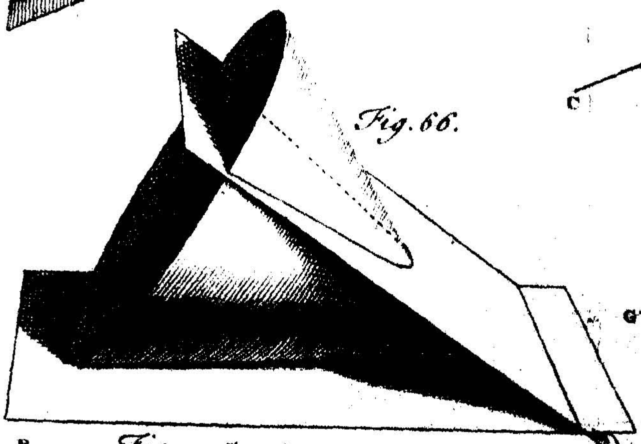
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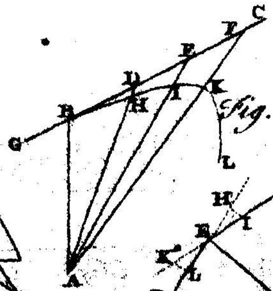
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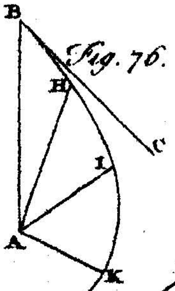
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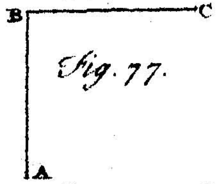
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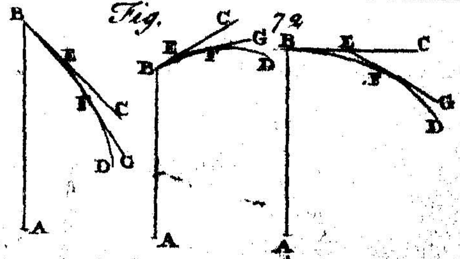
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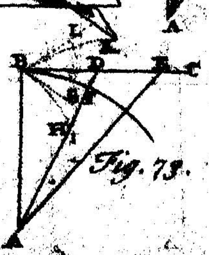
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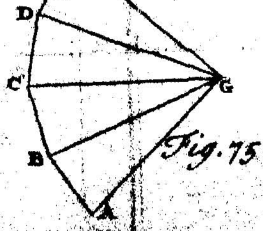
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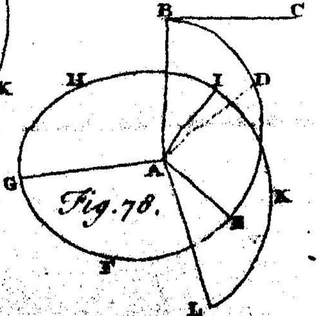
Fig. 78.

of the line AB into the line BC. The body after this impulſe, while left to itſelf, will continue moving in the line BC. At C let the body receive another impulſe directed towards the ſame point F, of ſuch ſtrength, as to turn the body from the line BC into the line CD. At D let the body by another impulſe, directed likewiſe to the point F, be turned out of the line CD into DE. And at E let another impulſe, directed toward the point F, turn the body from the line DE into EA. Thus we ſee how a body may be carried through the figure ABCDE by certain impulſes directed always toward the ſame center, only by their acting on the body at proper intervals, and with due degrees of ſtrength.

23. BUT farther, when the body is come to the point A, if it there receive another impulſe directed like the reſt toward the point F, and of ſuch a degree of ſtrength as to turn the body into the line AB, wherein it firſt moved; I ſay that the body ſhall return into this line with the ſame velocity, as it had at firſt.

24. Let AB be prolonged beyond B at pleaſure, ſuppoſe to G; and from G let GH be drawn, which if produced ſhould always continue equidiſtant from BF, or, according to the more uſual phraſe, let GH be drawn parallel to BF. Then it appears, from what has been ſaid upon the ſecond law of motion[a], that in the time, wherein the body would have moved from B to G, had it not received a new impulſe in B, by the means of that impulſe it will have acquired a velocity, which will carry it from B to H. After the ſame manner, if CI be

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[a] Ch. 2. § 22.

taken equal to B H, and I K be drawn equidiſtant from or parallel to C F; the body will have moved from C to K with the velocity, which it has in the line C D, in the ſame time, as it would have employed in moving from C to I with the velocity, it had in the line B C. Therefore ſince C I and B H are equal, the body will move through C K in the ſame time, as it would have taken up in moving from B to G with the original velocity, wherewith it moved through the line A B. Again, D L being taken equal to C K and L M drawn parallel to D F; for the ſame reaſon as before the body will move through D M with the velocity, which it has in the line D E, in the ſame time, as it would imploy in moving through B G with its original velocity. In the laſt place, if E N be taken equal to D M, and N O be drawn parallel to E F; likewiſe if A P be taken equal to E O, and P Q be drawn parallel to A F: then the body with the velocity, wherewith it returns into the line A B, will paſs through A Q in the ſame time, as it would have imployed in paſſing through B G with its original velocity. Now as all this follows directly from what has above been delivered, concerning the effect of oblique impulſes impreſſed upon bodies in motion; ſo we muſt here obſerve farther, that it can be proved by geometry, that A Q will always be equal to B G. The proof of this I am obliged, from the nature of my preſent deſign, to omit; but this geometrical propoſition being granted, it follows, that the body has returned into the line A B with the velocity, which it had, when it firſt moved in that line; for the velocity, with which it returns into the line A B, will carry it over the line A Q in the ſame time, as would have been taken up in its paſſing over an equal line BG with the original velocity.

25. Thus we have found, how a body may be carried round the figure ABCDE by the action of certain impulſes upon it, which ſhould all be pointed toward one center. And we likewiſe ſee, that when the body is brought back again to the point, whence it firſt ſet out; if it there meet with an impulſe ſufficient to turn it again into the line, wherein it moved at firſt, its original velocity will be again reſtored; and by the repetition of the ſame impulſes, the body will be carried again in the ſame round. Therefore if theſe impulſes, which act on the body at the points B, C, D, E, and A, continue always the ſame, the body will make round this figure innumerable revolutions.

26. The proof, which we have here made uſe of, holds the ſame in any number of ſtraight lines, whereof the figure ABD ſhould be compoſed; and therefore by the method of reaſoning referred to above [a] we are to conclude, that what has here been ſaid upon this rectilinear figure, will remain true, if this figure were changed into one of a continued curvature, and inſtead of diſtinct impulſes acting by intervals at the angles of this figure, we had a continual centripetal force. We have therefore ſhewn, that a body may be carried round in any curve figure ABC (fig. 82.) which ſhall every where be concave towards any one point as D, by the continual action

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[a] § 12.

of a centripetal power directed to that point, and when it is returned to the point, from whence it ſet out, it ſhall recover again the velocity, with which it departed from that point. It is not indeed always neceſſary, that it ſhould return again into its firſt courſe; for the curve line may have ſome ſuch figure as the line ABCDBE in fig. 83. In this curve line, if the body ſet out from B in the direction BF, and moved through the line BCD, till it returned to B; here the body would not enter again into the line BCD, becauſe the two parts BD and BC of the curve line make an angle at the point B: ſo that the centripetal power, which at the point B could turn the body from the line BF into the curve, will not be able to turn the body into the line BC from the direction, in which it returns to the point B; a forceable impulſe muſt be given the body in the point B to produce that effect.

27. If at the point B, whence the body ſets out, the curve line return into it ſelf (as in fig. 82;) then the body, upon its arrival again at B, may return into its former courſe, and thus make an endleſs circuit about the center of the centripetal power.

28. What has here been ſaid, I hope, will in ſome meaſure enable my readers to form a juſt idea of the nature of theſe centripetal motions.

29. I have not attempted to ſhew, how to find particularly, what kind of centripetal force is neceſſary to carry a body in any curve line propoſed. This is to be deduced from the degree

gree of curvature, which the figure has in each point of it, and requires a long and complex mathematical reaſoning. However I ſhall ſpeak a little to the firſt propoſition, which Sir ISAAC NEWTON lays down for this purpoſe. By this propoſition, when a body is found moving in a curve line, it may be known, whether the body be kept in its courſe by a power always pointed toward the ſame center; and if it be ſo, where that center is placed. The propoſition is this: that if a line be drawn from ſome fixed point to the body, and remaining by one extream united to that point, it be carried round along with the body; then, if the power, whereby the body is kept in its courſe, be always pointed to this fixed point as a center, this line will move over equal ſpaces in equal portions of time. Suppoſe a body were moving through the curve line ABCD (in fig. 84.) and paſſed over the arches AB, BC, CD in equal portions of time; then if a point, as E, can be found, from whence the line EA being drawn to the body in A, and accompanying the body in its motion, it ſhall make the ſpaces EAB, EBC, and ECD equal, over which it paſſes, while the body deſcribes the arches AB, BC, and CD: and if this hold the ſame in all other arches, both great and ſmall, of the curve line ABCD, that theſe ſpaces are always equal, where the times are equal; then is the body kept in this line by a power always pointed to E as a center.

30. THE principle, upon which Sir ISAAC NEWTON has demonſtrated this, requires but ſmall skill in geometry to comprehend. I ſhall therefore take the liberty to cloſe the pre-

of a centripetal power directed to that point, and when it is returned to the point, from whence it ſet out, it ſhall recover again the velocity, with which it departed from that point. It is not indeed always neceſſary, that it ſhould return again into its firſt courſe; for the curve line may have ſome ſuch figure as the line ABCDBE in fig. 83. In this curve line, if the body ſet out from B in the direction BF, and moved through the line BCD, till it returned to B; here the body would not enter again into the line BCD, becauſe the two parts BD and BC of the curve line make an angle at the point B: ſo that the centripetal power, which at the point B could turn the body from the line BF into the curve, will not be able to turn the body into the line BC from the direction, in which it returns to the point B; a forceable impulſe muſt be given the body in the point B to produce that effect.

27. IF at the point B, whence the body ſets out, the curve line return into it ſelf (as in fig. 82;) then the body, upon its arrival again at B, may return into its former courſe, and thus make an endleſs circuit about the center of the centripetal power.

28. WHAT has here been ſaid, I hope, will in ſome meaſure enable my readers to form a juſt idea of the nature of theſe centripetal motions.

29. I HAVE not attempted to ſhew, how to find particularly, what kind of centripetal force is neceſſary to carry a body in any curve line propoſed. This is to be deduced from the degree

gree of curvature, which the figure has in each point of it, and requires a long and complex mathematical reaſoning. However I ſhall ſpeak a little to the firſt propoſition, which Sir ISAAC NEWTON lays down for this purpoſe. By this propoſition, when a body is found moving in a curve line, it may be known, whether the body be kept in its courſe by a power always pointed toward the ſame center; and if it be ſo, where that center is placed. The propoſition is this: that if a line be drawn from ſome fixed point to the body, and remaining by one extream united to that point, it be carried round along with the body; then, if the power, whereby the body is kept in its courſe, be always pointed to this fixed point as a center, this line will move over equal ſpaces in equal portions of time. Suppoſe a body were moving through the curve line ABCD (in fig. 84.) and paſſed over the arches AB, BC, CD in equal portions of time; then if a point, as E, can be found, from whence the line EA being drawn to the body in A, and accompanying the body in its motion, it ſhall make the ſpaces EAB, EBC, and ECD equal, over which it paſſes, while the body deſcribes the arches AB, BC, and CD: and if this hold the ſame in all other arches, both great and ſmall, of the curve line ABCD, that theſe ſpaces are always equal, where the times are equal; then is the body kept in this line by a power always pointed to E as a center.

30. THE principle, upon which Sir ISAAC NEWTON has demonſtrated this, requires but ſmall skill in geometry to comprehend. I ſhall therefore take the liberty to cloſe the pre-

ſent chapter with an explication of it; becauſe ſuch an example will give the cleareſt notion of our author's method of applying mathematical reaſoning to theſe philoſophical ſubjects.

31. He reaſons thus. Suppoſe a body ſet out from the point A (in fig. 85.) to move in the ſtraight line AB; and after it had moved for ſome time in that line, it were to receive an impulſe directed to ſome point as C. Let it receive that impulſe at D; and thereby be turned into the line DE; and let the body after this impulſe take the ſame length of time in paſſing from D to E, as it imployed in the paſſing from A to D. Then the ſtraight lines CA, CD, and CE being drawn, Sir Isaac Newton proves, that the ~~and~~ triangular ſpaces CAD and CDE are equal. This he does in the following manner.

32. Let EF be drawn parallel to CD. Then, from what has been ſaid upon the ſecond law of motion [a], it is evident, that ſince the body was moving in the line AB, when it received the impulſe in the direction DC; it will have moved after that impulſe through the line DE in the ſame time, as it would have taken up in moving through DF, provided it had received no diſturbance in D. But the time of the body's moving from D to E is ſuppoſed to be equal to the time of its moving through AD; therefore the time, which the body would have imployed in moving through DF, had it not been diſturbed in D, is equal to the time, wherein it moved through AD: conſequently DF is equal in length to AD; for if the

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[a] Ch. 1. ſect. 21, 22.

body had gone on to move through the line A B without interruption, it would have moved through all parts thereof with the ſame velocity, and have paſſed over equal parts of that line in equal portions of time. Now C F being drawn, ſince A D and D F are equal, the triangular ſpace C D F is equal to the triangular ſpace C A D. Farther, the line E F being parallel to C D, it is proved by EUCLID, that the triangle C E D is equal to the triangle C F D [a]: therefore the triangle C E D is equal to the triangle C A D.

33. AFTER the ſame manner, if the body receive at E another impulſe directed toward the point C, and be turned by that impulſe into the line E G; if it move afterwards from E to G in the ſame ſpace of time, as was taken up by its motion from D to E, or from A to D; then C G being drawn, the triangle C E G is equal to C D E. A third impulſe at G directed as the two former to C, whereby the body ſhall be turned into the line G H, will have alſo the like effect with the reſt. If the body move over G H in the ſame time, as it took up in moving over E G, the triangle C G H will be equal to the triangle C E G. Laſtly, if the body at H be turned by a freſh impulſe directed toward C into the line H I, and at I by another impulſe directed alſo to C be turned into the line I K; and if the body move over each of the lines H I, and I K in the ſame time, as it imployed in moving over each of the preceding lines A D, D E, E G, and G H: then each of the triangles C H I, and C I K will be equal to each of the preceding. Like-

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[a] Elem, Book I. p. 37.

wiſe as the time, in which the body moves over ADE, is equal to the time of its moving over EGH, and to the time of its moving over HIK; the ſpace CADE will be equal to the ſpace CEGH, and to the ſpace CHIK. In the ſame manner as the time, in which the body moved over ADEG is equal to the time of its moving over GHIK, ſo the ſpace CADEG will be equal to the ſpace CGHIK.

34. From this principle Sir Isaac Newton demonſtrates the propoſition mentioned above, by that method of arguing introduced by him into geometry, whereof we have before taken notice [a], by making according to the principles of that method a tranſition from this incurvated figure compoſed of ſtraight lines, to a figure of continued curvature; and by ſhewing, that ſince equal ſpaces are deſcribed in equal times in this preſent figure compoſed of ſtraight lines, the ſame relation between the ſpaces deſcribed and the times of their deſcription will alſo have place in a figure of one continued curvature. He alſo deduces from this propoſition the reverſe of it; and proves, that whenever equal ſpaces are continually deſcribed; the body is acted upon by a centripetal force directed to the center, at which the ſpaces terminate.

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[a] § 12.

CHAP. IV.

Of the RESISTANCE of FLUIDS.

BEFORE the cauſe can be diſcovered, which keeps the planets in motion, it is neceſſary firſt to know, whether the ſpace, wherein they move, is empty and void, or filled with any quantity of matter. It has been a prevailing opinion, that all ſpace contains in it matter of ſome kind or other; ſo that where no ſenſible matter is found, there was yet a ſubtle fluid ſubſtance by which the ſpace was filled up; even ſo as to make an abſolute plenitude. In order to examine this opinion, Sir ISAAC NEWTON has largely conſidered the effects of fluids upon bodies moving in them.

2. THESE effects he has reduced under theſe three heads. In the firſt place he ſhews how to determine in what manner the reſiſtance, which bodies ſuffer, when moving in a fluid, gradually increaſes in proportion to the ſpace, they deſcribe in any fluid; to the velocity, with which they deſcribe it; and to the time they have been in motion. Under the ſecond head he conſiders what degree of reſiſtance different bodies moving in the ſame fluid undergo, according to the different proportion between the denſity of the fluid and the denſity of the body. The denſities of bodies, whether fluid or ſolid, are meaſured by the quantity of matter, which is comprehended under the ſame magnitude; that body being

the moſt denſe or compact, which under the ſame bulk contains the greateſt quantity of ſolid matter, or which weighs moſt, the weight of every body being obſerved above to be proportional to the quantity of matter in it [a]. Thus water is more denſe than cork or wood, iron more denſe than water, and gold than iron. The third particular Sir Iſ. Newton conſiders concerning the reſiſtance of fluids is the influence, which the diverſity of figure in the reſiſted body has upon its reſiſtance.

3. For the more perfect illuſtration of the firſt of theſe heads, he diſtinctly ſhews the relation between all the particulars ſpecified upon three different ſuppoſitions. The firſt is, that the ſame body be reſiſted more or leſs in the ſimple proportion to its velocity; ſo that if its velocity be doubled, its reſiſtance ſhall become ~~threefold~~. The ſecond is of the reſiſtance increaſing in the duplicate proportion of the velocity; ſo that, if the velocity of a body be doubled, its reſiſtance ſhall be rendered four times; and if the velocity be trebled, nine times as great as at firſt. But what is to be underſtood by duplicate proportion has been already explained [b]. The third ſuppoſition is, that the reſiſtance increaſes partly in the ſingle proportion of the velocity, and partly in the duplicate proportion thereof.

4. In all theſe ſuppoſitions, bodies are conſidered under two reſpects, either as moving, and oppoſing themſelves

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[a] Ch. 1. § 24.

[b] Ch. 2. ſect. 17.

againſt the fluid by that power alone, which is eſſential to them, of reſiſting to the change of their ſtate from reſt to motion, or from motion to reſt, which we have above called their power of inactivity; or elſe, as deſcending or aſcending, and ſo having the power of gravity combined with that other power. Thus our author has ſhewn in all thoſe three ſuppoſitions, in what manner bodies are reſiſted in an uniform fluid, when they move with the aforeſaid progreſſive motion [a]; and what the reſiſtance is, when they aſcend or deſcend perpendicularly [b]. And if a body aſcend or deſcend obliquely, and the reſiſtance be ſingly proportional to the velocity, it is ſhewn how the body is reſiſted in a fluid of an uniform denſity, and what line it will deſcribe [c], which is determined by the meaſurement of the hyperbola, and appears to be no other than that line, firſt conſidered in particular by Dr. BARROW [d], which is now commonly known by the name of the logarithmical curve. In the ſuppoſition that the reſiſtance increaſes in the duplicate proportion of the velocity, our author has not given us the line which would be deſcribed in an uniform fluid; but has inſtead thereof diſcuſſed a problem, which is in ſome ſort the reverſe; to find the denſity of the fluid at all altitudes, by which any given curve line may be deſcribed; which problem is ſo treated by him, as to be applicable to any kind of reſiſtance whatever [e]. But here not unmindful of practice, he ſhews that a body in a fluid of uniform denſity, like the air, will deſcribe a line, which approaches towards an hyperbola; that is, its motion will be nearer to that curve line than to the parabola. And conſequent upon this remark, he ſhews how to determine this hyperbola by experiment, and briefly reſolves the chief of thoſe problems relating to projectiles, which are in uſe in the art of gunnery, in this curve[a]; as Torricelli and others have done in the parabola[b], whoſe inventions have been explained at large above

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[a] Newt. Princ. L. II. prop. 2; 5, 6, 7; 11, 12.

[b] Prop. 3; 8, 9; 13, 14.

[c] Prop. 4.

[d] Prælect. Geometr. pag. 123.

[e] Newton. Princ. Lib. II. prop. 10.

5. Our author has alſo handled diſtinctly that particular ſort of motion, which is deſcribed by pendulums[d]; and has likewiſe conſidered ſome few caſes of bodies moving in reſiſting fluids round a center, to which they are impelled by a centripetal force, in order to give an idea of thoſe kinds of motions[e]

6. The treating of the reſiſtance of pendulums has given him an opportunity of inſerting into another part of his work ſome ſpeculations upon the motions of them without reſiſtance, which have a very peculiar elegance; wherein he treats of them as moved by a gravitation acting in the law, which he ſhews to belong to the earth below its ſurface[f]; performing in this kind of gravitation, where the force is proportional to the diſtance from the center, all that Huygens had before done in the common ſuppoſition of its being uniform, and acting in parallel lines[g].

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[a] Newton. Princ. Lib. II prop 10. in ſchol.

[b] Torricelli de motu gravium.

[c] Ch. 2. § 85, &c.

[d] Newt. Princ L. II. ſect. 6.

[e] L. II. ſect. 4.

[f] See B. II. Ch 6. § 7. of this treatiſe.

[g] Lib. I. ſect. 10.

7. HUYGENS at the end of his treatiſe of the cauſe of gravity[a] informs us, that he likewiſe had carried his ſpeculations on the firſt of theſe ſuppoſitions, of the reſiſtance in fluids being proportional to the velocity of the body, as far as our author. But finding by experiment that the ſecond was more conformable to nature, he afterwards made ſome progreſs in that, till he was ſtopt, by not being able to execute to his wiſh what related to the perpendicular deſcent of bodies; not obſerving that the meaſurement of the curve line, he made uſe of to explain it by, depended on the hyperbola. Which overſight may well be pardoned in that great man, conſidering that our author had not been pleaſed at that time to communicate to the publick his admirable diſcourſe of the QUADRATURE OF MEASUREMENT OF CURVE LINES, with which he has ſince obliged the world: for without the uſe of that treatiſe, it is I think no injury even to our author's unparalleled abilities to believe, it would not have been eaſy for himſelf to have ſucceeded ſo happily in this and many other parts of his writings.

8. WHAT HUYGENS found by experiment, that bodies were in reality reſiſted in the duplicate proportion of their velocity, agrees with the reaſoning of our author[b], who diſtinguiſhes the reſiſtance, which fluids give to bodies by the tenacity of their parts, and the friction between them and the body, from that, which ariſes from the power of inactivity, with which the conſtituent particles of fluids are endued like all

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[a] De la Peſanteur, pag. 169, and the following.

[b] Newton. Princ. L. II. prop. 4. ſchol.

other portions of matter, by which power the particles of fluids like other bodies make reſiſtance againſt being put into motion.

9. THE reſiſtance, which ariſes from the friction of the body againſt the parts of the fluid, muſt be very inconſiderable; and the reſiſtance, which follows from the tenacity of the parts of fluids, is not uſually very great, and does not depend much upon the velocity of the body in the fluid; for as the parts of the fluid adhere together with a certain degree of force, the reſiſtance, which the body receives from thence, cannot much depend upon the velocity, with which the body moves; but like the power of gravity, its effect muſt be proportional to the time of its acting. This the reader may find farther explained by Sir ISAAC NEWTON himſelf in the poſtſcript to a diſcourſe publiſhed by me in THE PHILOSOPHICAL TRANSACTIONS, N° 371. The principal reſiſtance, which moſt fluids give to bodies, ariſes from the power of inactivity in the parts of the fluids, and this depends upon the velocity, with which the body moves, on a double account. In the firſt place, the quantity of the fluid moved out of place by the moving body in any determinate ſpace of time is proportional to the velocity, wherewith the body moves; and in the next place, the velocity with which each particle of the fluid is moved, will alſo be proportional to the velocity of the body: therefore ſince the reſiſtance, which any body makes againſt being put into motion, is proportional both to the quantity of matter moved and the velocity it is moved with; the reſiſtance, which a fluid gives on this account, will be doubly increaſed with the increaſe of the velocity in the moving body;

that is, the reſiſtance will be in a two-fold or duplicate proportion of the velocity, wherewith the body moves through the fluid.

10. FARTHER it is moſt manifeſt, that this latter kind of reſiſtance increaſing with the increaſe of velocity, even in a greater degree than the velocity it ſelf increaſes, the ſwifter the body moves, the leſs proportion the other ſpecies of reſiſtance will bear to this: nay that this part of the reſiſtance may be ſo much augmented by a due increaſe of velocity, till the former reſiſtances ſhall bear a leſs proportion to this, than any that might be aſſigned. And indeed experience ſhews, that no other reſiſtance, than what ariſes from the power of inactivity in the parts of the fluid, is of moment, when the body moves with any conſiderable ſwiftneſs.

11. THERE is beſides theſe yet another ſpecies of reſiſtance, found only in ſuch fluids, as, like our air, are elaſtic. Elaſticity belongs to no fluid known to us beſide the air. By this property any quantity of air may be contracted into a leſs ſpace by a forcible preſſure, and as ſoon as the compreſſing power is removed, it will ſpring out again to its former dimenſions. The air we breath is held to its preſent denſity by the weight of the air above us. And as this incumbent weight, by the motion of the winds, or other cauſes, is frequently varied (which appears by the barometer;) ſo when this weight is greateſt, we breath a more denſe air than at other times. To what degree the air would expand it ſelf by its ſpring, if all preſſure were removed, is not

known, nor yet into how narrow a compaſs it is capable of being compreſſed. Mr. BOYLE found it by experiment capable both of expanſion and compreſſion to ſuch a degree, that he could cauſe a quantity of air to expand it ſelf over a ſpace ſome hundred thouſand times greater, than the ſpace to which he could confine the ſame quantity [a]. But I ſhall treat more fully of this ſpring in the air hereafter [b]. I am now only to conſider what reſiſtance to the motion of bodies ariſes from it.

12. BUT before our author ſhews in what manner this cauſe of reſiſtance operates, he propoſes a method, by which fluids may be rendered elaſtic, demonſtrating that if their particles be provided with a power of repelling each other, which ſhall exert it ſelf with degrees of ſtrength reciprocally proportional to the diſtances between the centers of the particles; that then ſuch fluids will obſerve the ſame rule in being compreſſed, as our air does, which is this, that the ſpace, into which it yields upon compreſſion, is reciprocally proportional to the compreſſing weight [c]. The term reciprocally proportional has been explained above [d]. And if the centrifugal force of the particles acted by other laws, ſuch fluids would yield in a different manner to compreſſion [e].

13. WHETHER the particles of the air be endued with ſuch a power, by which they can act upon each other out of contact, our author does not determine; but leaves that

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[a] See his Tract on the admirable rarifaction of the air.

[b] Book II. Ch. 6.

[c] Princ philoſ. Lib. II. prop. 23.

[d] Book I. Ch. 2. § 30

[e] Princ. philoſ. Lib. II. prop. 23. in ſchol.

to future examination, and to be diſcuſſed by philoſophers. Only he takes occaſion from hence to conſider the reſiſtance in elaſtic fluids, under this notion; making remarks, as he paſſes along, upon the differences, which will ariſe, if their elaſticity be derived from any other fountain [a]. And this, I think, muſt be confeſſed to be done by him with great judgment; for this is far the moſt reaſonable account, which has been given of this ſurprizing power, as muſt without doubt be freely acknowledged by any one, who in the leaſt conſiders the inſufficiency of all the other conjectures, which have been framed; and alſo how little reaſon there is to deny to bodies other powers, by which they may act upon each other at a diſtance, as well as that of gravity; which we ſhall hereafter ſhew to be a property univerſally belonging to all the bodies of the univerſe, and to all their parts [b]. Nay we actually find in the loadſtone a very apparent repelling, as well as an attractive power. But of this more in the concluſion of this diſcourſe.

14. By theſe ſteps our author leads the way to explain the reſiſtance, which the air and ſuch like fluids will give to bodies by their elaſticity; which reſiſtance he explains thus. If the elaſtic power of the fluid were to be varied ſo, as to be always in the duplicate proportion of the velocity of the reſiſted body, it is ſhewn that then the reſiſtance derived from the elaſticity, would increaſe in the duplicate proportion of the velocity; in ſo much that the

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[a] Princ. philoſ. Lib. II. prop. 33. coroll.

[b] Lib. II. Ch. 5.

whole reſiſtance would be in that proportion, excepting only that ſmall part, which ariſes from the friction between the body and the parts of the fluid. From whence it follows, that becauſe the elaſtic power of the ſame fluid does in truth continue the ſame, if the velocity of the moving body be diminiſhed, the reſiſtance from the elaſticity, and therefore the whole reſiſtance, will decreaſe in a leſs proportion, than the duplicate of the velocity; and if the velocity be increaſed, the reſiſtance from the elaſticity will increaſe in a leſs proportion, than the duplicate of the velocity, that is in a leſs proportion, than the reſiſtance made by the power of inactivity of the parts of the fluid. And from this foundation is raiſed the proof of a property of this reſiſtance, given by the elaſticity in common with the others from the tenacity and friction of the parts of the fluid; that the velocity may be increaſed, till this reſiſtance from the fluid's elaſticity ſhall bear no conſiderable proportion to that, which is produced by the power of inactivity thereof [a]. From whence our author draws this concluſion; that the reſiſtance of a body, which moves very ſwiftly in an elaſtic fluid, is near the ſame, as if the fluid were not elaſtic; provided the elaſticity ariſes from the centrifugal power of the parts of the medium, as before explained, eſpecially if the velocity be ſo great, that this centrifugal power ſhall want time to exert it ſelf [b]. But it is to be obſerved, that in the proof of all this our author proceeds upon the ſuppoſition of this centrifugal power in the parts of the fluid; but if the elaſticity be cauſed by the expanſion of the parts in the manner of wool compreſſed, and ſuch like bodies, by which the parts of the fluid will be in ſome meaſure entangled together, and their motion be obſtructed, the fluid will be in a manner tenacious, and give a reſiſtance upon that account over and above what depends upon its elaſticity only [a]; and the reſiſtance derived from that cauſe is to be judged of in the manner before ſet down.

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[a] Ibid. Prop. 33. coroll. 2.

[b] Ibid. coroll. 3.

15. IT is now time to paſs to the ſecond part of this theory; which is to aſſign the meaſure of reſiſtance, according to the proportion between the denſity of the body and the denſity of the fluid. What is here to be underſtood by the word denſity has been explained above [b]. For this purpoſe as our author before conſidered two diſtinct caſes of bodies moving in mediums; one when they oppoſed themſelves to the fluid by their power of inactivity only, and another when by aſcending or deſcending their weight was combined with that other power: ſo likewiſe, the fluids themſelves are to be regarded under a double capacity; either as having their parts at reſt, and diſpoſed freely without reſtraint, or as being compreſſed together by their own weight, or any other cauſe.

16. IN the firſt caſe, if the parts of the fluid be wholly diſingaged from one another, ſo that each particle is at liberty to move all ways without any impediment, it is ſhewn, that if a globe move in ſuch a fluid, and the globe and par-

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[a] Vid. ibid. coroll. 6.

[b] In § 2.

ticles of the fluid are endued with perfect elaſticity; ſo that as the globe impinges upon the particles of it, they ſhall bound off and ſeparate themſelves from the globe, with the ſame velocity, with which the globe ſtrikes upon them; then the reſiſtance, which the globe moving with any known velocity ſuffers, is to be thus determined. From the velocity of the globe, the time, wherein it would move over two third parts of its own diameter with that velocity, will be known. And ſuch proportion as the denſity of the fluid bears to the denſity of the globe, the ſame the reſiſtance given to the globe will bear to the force, which acting, like the power of gravity, on the globe without intermiſſion during the ſpace of time now mentioned, would generate in the globe the ſame degre of motion, as that wherewith it moves in the fluid[a]. But if neither the globe nor the particles of the fluid be elaſtic, ſo that the particles, when the globe ſtrikes againſt them, do not rebound from it, then the reſiſtance will be but half ſo much[b]. Again, if the particles of the fluid and the globe are imperfectly elaſtic, ſo that the particles will ſpring from the globe with part only of that velocity wherewith the globe impinges upon them; then the reſiſtance will be a mean between the two preceding caſes, approaching nearer to the firſt or ſecond, according as the elaſticity is more or leſs[c].

17. THE elaſticity, which is here aſcribed to the particles of the fluid, is not that power of repelling one another,

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[a] Princ. philoſ. Lib. II. Prop. 35.

[b] Ibid.

[c] Id.

when out of contact, by which, as has before been mentioned, the whole fluid may be rendred elaſtic; but ſuch an elaſticity only, as many ſolid bodies have of recovering their figure, whenever any forcible change is made in it, by the impulſe of another body or otherwiſe. Which elaſticity has been explained above at large [a]

18. THIS is the caſe of diſcontinued fluids, where the body, by preſſing againſt their particles, drives them before itſelf, while the ſpace behind the body is left empty. But in fluids which are compreſſed, ſo that the parts of them removed out of place by the body reſiſted immediately retire behind the body, and fill that ſpace, which in the other caſe is left vacant, the reſiſtance is ſtill leſs; for a globe in ſuch a fluid which ſhall be free from all elaſticity, will be reſiſted but half as much as the leaſt reſiſtance in the former caſe [b]. But by elaſticity I now mean that power, which renders the whole fluid ſo; of which if the compreſſed fluid be poſſeſſed, in the manner of the air, then the reſiſtance will be greater than by the foregoing rule; for the fluid being capable in ſome degree of condenſation, it will reſemble ſo far the caſe of uncompreſſed fluids [c]. But, as has been before related, this difference is moſt conſiderable in ſlow motions.

19. IN the next place our author is particular in determining the degrees of reſiſtance accompanying bodies of different figures; which is the laſt of the three heads, we

[a] h. 1. § 29.

[b] Princ. philoſ. Lib. II. Prop. 38, compared with coroll. 1. of prop. 35.

[c] L. II. Lem. 7. ſchol. pag. 341.

divided the whole diſcourſe of reſiſtance into. And in this diſquiſition he finds a very ſurprizing and unthought of difference, between free and compreſſed fluids. He proves, that in the former kind, a globe ſuffers but half the reſiſtance, which the cylinder, that circumſcribes the globe, will do, if it move in the direction of its axis[a]. But in the latter he proves, that the globe and cylinder are reſiſted alike[b]. And in general, that let the ſhape of bodies be ever ſo different, yet if the greateſt ſections of the bodies perpendicular to the axis of their motion be equal, the bodies will be reſiſted equally[c].

20. PURSUANT to the difference found between the reſiſtance of the globe and cylinder in rare and uncompreſſed fluids, our author gives us the reſult of ſome other inquiries of the ſame nature. Thus of all the fruſtums of a cone, that can be deſcribed upon the ſame baſe and with the ſame altitude, he ſhews how to find that, which of all others will be the leaſt reſiſted, when moving in the direction of its axis[d]. And from hence he draws an eaſy method of altering the figure of any ſpheroidical ſolid, ſo that its capacity may be enlarged, and yet the reſiſtance of it diminiſhed[e]: a note which he thinks may not be uſeleſs to ſhipwrights. He concludes with determining the ſolid, which will be reſiſted the leaſt that is poſſible, in theſe diſcontinued fluids[f].

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[a] Lib. II. Prop. 34.

[b] Lib. II. Lem. 7. p. 341.

[c] Schol. to Lem. 7.

[d] Prop. 34. ſchol.

[e] Ibid.

[f] Ibid.

21. THAT I may here be underſtood by readers unacquainted with mathematical terms, I ſhall explain what I mean by a fruſtum of a cone, and a ſpheroidical ſolid. A cone has been defined above. A fruſtum is what remains, when part of the cone next the vertex is cut away by a ſection parallel to the baſe of the cone, as in fig. 86. A ſpheroid is produced from an ellipſis, as a ſphere or globe is made from a circle. If a circle turn round on its diameter, it deſcribes by its motion a ſphere; ſo if an ellipſis (which figure has been defined above, and will be more fully explained hereafter[a]) be turned round either upon the longeſt or ſhorteſt line, that can be drawn through the middle of it, there will be deſcribed a kind of oblong or flat ſphere, as in fig. 87. Both theſe figures are called ſpheroids, and any ſolid reſembling theſe I here call ſpheroidical.

22. IF it ſhould be asked, how the method of altering ſpheroidical bodies, here mentioned, can contribute to the facilitating a ſhip's motion, when I juſt above affirmed, that the figure of bodies, which move in a compreſſed fluid not elaſtic, has no relation to the augmentation or diminution of the reſiſtance; the reply is, that what was there ſpoken relates to bodies deep immerged into ſuch fluids, but not of thoſe, which ſwim upon the ſurface of them; for in this latter caſe the fluid, by the appulſe of the anterior parts of the body, is raiſed above the level of the ſurface, and behind the body is ſunk ſomewhat below; ſo

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[a] Book II. Ch. 1. § 6.

that by this inequality in the ſuperficies of the fluid, that part of it, which at the head of the body is higher than the fluid behind, will reſiſt in ſome meaſure after the manner of diſcontinued fluids[a], analogous to what was before obſerved to happen in the air through its elaſticity, though the body be ſurrounded on every ſide by it[b]. And as far as the power of theſe cauſes extends, the figure of the moving body affects its reſiſtance; for it is evident, that the figure, which preſſes leaſt directly againſt the parts of the fluid, and ſo raiſes leaſt the ſurface of a fluid not elaſtic, and leaſt compreſſes one that is elaſtic, will be leaſt reſiſted.

23. THE way of collecting the difference of the reſiſtance in rare fluids, which ariſes from the diverſity of figure, is by conſidering the different effect of the particles of the fluid upon the body moving againſt them, according to the different obliquity of the ſeveral parts of the body upon which they reſpectively ſtrike; as it is known, that any body impinging againſt a plane obliquely, ſtrikes with a leſs force, than if it fell upon it perpendicularly; and the greater the obliquity is, the weaker is the force. And it is the ſame thing, if the body be at reſt, and the plane move againſt it[c].

24. THAT there is no connexion between the figure of a body and its reſiſtance in compreſſed fluids, is proved thus. Suppoſe ABCD (in fig. 88.) to be a canal, having ſuch a fluid, water for inſtance, running through it with an equable

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[a] Vid. Newt. princ. in ſchol. to Lem. 7. of Lib. II. pag. 341.

[b] Sect. 17. of this chapter.

[c] See Princ. philoſ. Lib. II. prop. 34.

velocity;